TENSION OF A PLATE CONTAINING A RECTILINEAR CUT WITH HINGED RIMS

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Let us consider an infinite isotropic plate of thickness 2h, containing a rectilinear through cut of length 2ℓ (Fig. 1). The sides of the cut $y = \pm 0$ are joined along the edges with hinges in one of the faces $z = sh$ ($s = -1$ or $s = +1$). The faces of the plate and the edges of the cut are free of external loads. We study the effect of the cut on the stressed state of the plate caused by forces $n = \text{const}$ distributed uniformly at infinity.

Since the system is asymmetric about the middle of the plane of the plate, a local bend should be expected in the proximity of a stress concentrator. Besides the equation of the generalized plane stressed state

$$\Delta\Delta\varphi = 0, \tag{1}$$

therefore, we also use the equation for the bending of plates in the Kirchhoff theory

$$\Delta\Delta w = 0 \tag{2}$$

to describe the elastic equilibrium state of the plate beyond the cut. Here φ is the Airy function, w is the deflection of the plate, and $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian.

For membrane forces and bending moments at infinity we require satisfaction of the conditions

$$N_x = N_{xy} = 0,\ N_y = n,\ M_x = M_{xy} = M_y = 0,\ x^2 + y^2 \to \infty. \tag{3}$$

Taking into account the symmetry of the problem about the x axis, we consider the boundary conditions at the cut.

In any cross section $x = \text{const} \in (-\ell, \ell)$ (Fig. 2), besides the opening of the cut [v], a jump in the angle of rotation of the normal $[\vartheta_y]$ appears in the middle of the plate surface. Here $\vartheta_y = \partial w/\partial y$, $[f] = f(x, +0) - f(x, -0)$. Following the hypotheses of Kirchhoff about a rigid normal element, from the conditions of continuity of the elastic displacements at the joined edges we obtain

$$[v] - sh[\vartheta_y] = 0,\ x \in (-l,\ l), \tag{4}$$

and at the ends of the cut we have

$$[v](\pm l) = 0,\ [\vartheta_y](\pm l) = 0. \tag{5}$$

The static diagram of the contact is shown in Fig. 3. We replace the unknown reaction R of the hinge with an equivalent system: membrane forces $N_y = R$ and bending moments $M_y = shR$. Eliminating R, we arrive at the condition

$$M_y - shN_y = 0,\ y = 0,\ x \in (-l,\ l). \tag{6}$$

Moreover, tangential and generalized shearing forces do not exist at the cut, i.e.,

$$N_{xy} = 0,\quad Q_y^* = 0,\quad y = 0,\quad x \in (-l, l). \tag{7}$$

We construct the solution of the problem (1)-(7) by using the method of integral equations. The general solution of Eqs. (1) and (2), satisfying conditions (3) and (7), is written as [1]

$$\varphi(x, y) = \varphi^0(x, y) + \frac{B}{8\pi}\int_{-l}^{l}[v](\xi)\left(2\ln r + 1 + 2\frac{y^2}{r^2}\right)d\xi,$$

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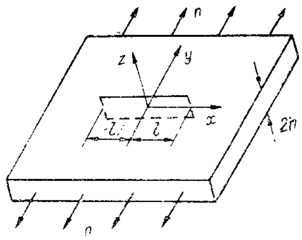

Fig. 1

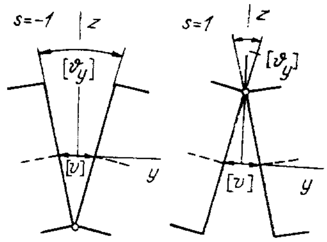

Fig. 2

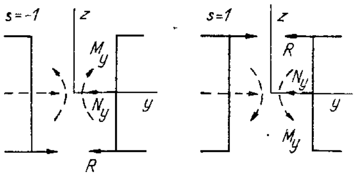

Fig. 3

$$w(x, y) = \frac{1}{8\pi} \int_{-l}^{l} [\vartheta_y](\xi) \left(2(1+\nu)\ln r + 1 + 3\nu + 2(1-\nu)\frac{y^2}{r^2} \right) d\xi, \tag{8}$$

where $\varphi^0(x, y) = ny^2/2$ is a function of the stresses in the tensioned plate without a cut, $r = \sqrt{(x-\xi)^2 + y^2}$, $B = 2Eh$, and E and ν are Young's modulus and Poisson's ratio of the material of the plate.

The integral representations, corresponding to Eqs. (8), of the normal forces and moments on the line of the cut in terms of the derivatives of unknown functions of the jump are given by the formulas.

$$\begin{aligned} N_y(x, 0) &= n + \frac{B}{4\pi} \int_{-l}^{l} [v]'(\xi) \frac{d\xi}{\xi - x}, \\ M_y(x, 0) &= -\frac{D}{4\pi} \int_{-l}^{l} [\vartheta_y]'(\xi) \frac{3 - 2\nu - \nu^2}{\xi - x}\, d\xi \end{aligned} \tag{9}$$

where $D = 2Eh^3/(3(1-\nu^2))$ is the cylindrical rigidity of the plate.

Substituting Eq. (9) into the boundary condition (6) and eliminating function $[\vartheta_y]$ by means of Eq. (4), we arrive at the integrodifferential equation.

$$\frac{\varkappa + 1}{\varkappa} \frac{B}{4\pi} \int_{-l}^{l} [v]'(\xi) \frac{d\xi}{\xi - x} = -n, \quad x \in (-l, l) \quad (\varkappa = 3(1+\nu)/(3+\nu)). \tag{10}$$

The solution of Eq. (10), which satisfies the first condition (5) is generally known: $[v] = (4n\kappa\sqrt{l^2 - x^2})/[B(1+\kappa)]$. The jump in the angle of rotation is determined from Eq. (4), $[\vartheta_y] = (4snh\sqrt{l^2 - x^2})/[D(3 - 2\nu - \nu^2)(1+\kappa)]$, and the unknown reactions at the cut are found from Eqs. (9):

$$N_y(x, 0) = n/(1+\varkappa), \ M_y(x, 0) = snh/(1+\varkappa), \ x \in (-l, l).$$

From the known jumps of the displacements and the angle of rotation of the normal we can determine the stress–strain state of the plate over the entire region on the basis of representations (8).

We point out one possible application of the results. The scheme described here simulates the problem of tension of a plate with a cut, covered on one side with a flexible film that deforms along with the plate. Let us assess how the hinged coupling of the edges of the cut affects the bearing capacity of the plate, basing ourselves on the energy concept of linear fracture mechanics.

The intensity factors of the forces and moments [2] in the neighborhood of the ends of the cut are calculated from the formulas

$$K_1 = -\frac{B}{4\sqrt{l}} \lim_{x\to l} \sqrt{l^2 - x^2}\,[v]' = \frac{\varkappa}{1+\varkappa} n\sqrt{l}, \quad K_3 = (3 - 2\nu - \nu^2)\frac{D}{4\sqrt{l}} \lim_{x\to l} \sqrt{l^2 - x^2}\,[\vartheta_y]' = -\frac{snh\sqrt{l}}{1+\varkappa}. \qquad (11)$$

The expression for the energy flux to the crack tip under combined tension and bending has the form [3] $G = \pi/4h^2E\{K_1^2 + \kappa(K_3/h)^2\}$. Assuming that as the crack grows the film and the adhesive bond remain undamaged, on the basis of Eqs. (11) we find $G = \pi \ell n^2/4h^2E \times \kappa/(1 + \kappa)$. We note that in the absence of a film $K_1' = n\sqrt{l}$, $K_3' = 0$, and $G' = \pi \ell n^2/(4h^2E)$.

In summary, the application of a flexible reinforcement to one face of a tensioned plate with a crack, within the framework of the formulation under consideration here, causes the energy flux to decrease by a factor of $G'/G = (\kappa + 1)/\kappa$ and the bearing capacity of the plate to increase by a factor of $\sqrt{(\varkappa + 1)/\varkappa} \approx 1.33\text{-}1.41$.

LITERATURE CITED

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